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Application of Feedback Linearization to Tracking and Almost Disturbance Decoupling Control of the AMIRA Ball and Beam System

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Abstract. This paper studies the tracking and almost disturbance decoupling problem of the nonlinear AMIRA ball and beam system based on the feedback linearization approach. The main contribution of this study is to construct a controller, under appropriate conditions, such that the resulting closed-loop system is valid for any initial condition and bounded tracking signal with the following characteristics: input-to-state stability with respect to disturbance inputs and almost disturbance decoupling. Two examples on the almost disturbance decoupling problem, which cannot be solved via Ref. 1, are proposed in this paper exploiting the fact that the tracking and the almost disturbance decoupling performances are easily achieved by our proposed approach.

Key Words. Almost disturbance decoupling, feedback linearization, composite Lyapunov approach, AMIRA ball and beam system, singularly-perturbed system.

1. Introduction

The well-known tasks of stabilization and tracking are important topics in the field of control. The tracking problem is generally more complicated than the stabilization problem for nonlinear control systems. For nonlinear systems, many approaches have been introduced including feedback

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linearization, variable structure control (sliding mode control), backstepping, regulation control, nonlinear H_∞ control, internal model principle, and H_∞ adaptive fuzzy control.

Recently, variable structure control has been introduced to deal with nonlinear systems (Ref. 2); however, chattering behavior, that may create unmodeled high-frequency due to discontinuous switching and imperfect implementation and may even drive the system to instability, is inevitable for a variable structure control scheme. Backstepping has been a powerful tool for synthesizing the controller for a class of nonlinear systems; however, a disadvantage with the backstepping approach is the complexity caused by repeated differentiations of some nonlinear functions (Refs. 3–4).

An output tracking approach utilizes the scheme of output regulation control (Ref. 5) in which the outputs are assumed to be excited by an exosystem; however, the nonlinear regulation problem requires achieving the difficult solution of a partial-differential algebraic equation. Another problem of output regulation control is that the exosystem states need to be switched to describe changes in the output and this creates transient tracking errors (Ref. 6). In general, the nonlinear H_∞ control must solve the Hamilton-Jacobi equation, which is a difficult nonlinear partial-differential equation (Refs. 7–9). Only for particular nonlinear systems we can derive a closed-form solution.

The control approach based on internal model principle converts the tracking problem to a nonlinear output regulation problem. This approach depends on solving a first-order partial-differential equation of the center manifold (Ref. 5). For some special nonlinear systems and desired trajectories, asymptotic solutions of the above equation via ordinary differential equations have been developed (Refs. 10–11). Recently, H_∞ adaptive fuzzy control has been proposed to deal systematically with nonlinear systems (Ref. 12). The drawback with H_∞ adaptive fuzzy control is that the complex parameter update law makes this approach impractical.

During the past decade, significant progress has been made in the research of control approaches for nonlinear systems based on feedback linearization theory (Refs. 2 and 13–15). Moreover, the feedback linearization approach has been applied successfully to address many real control problems. These include the control of an electromagnetic suspension system (Ref. 19), pendulum system (Ref. 17), spacecraft (Ref. 18), electro-hydraulic servosystem (Ref. 19), car-pole system (Ref. 20), and bank-to-turn missile system (Ref. 21).

The almost disturbance decoupling problem, that is, the design of a controller which attenuates the effect of the disturbance on the output terminal to an arbitrary degree of accuracy, was developed originally for linear control systems (Ref. 1) and nonlinear control systems (Ref. 22).

Henceforward, the problem has attracted considerable attention and many significant results have been obtained for both linear and nonlinear control systems (Refs. 23–25). Reference 1 shows that, for nonlinear SISO systems, the almost disturbance decoupling problem may not be solvable, as the following examples show:

(i) $\dot{x}_1(t) = \tan^{-1} x_2 + \theta(t),\ \dot{x}_2(t) = u,$
$y = x_1;$

(ii) $\dot{x}_1(t) = x_2 + \theta_1(t),\ \dot{x}_2(t) = x_2^3\theta_2(t) + u,$
$y = x_1.$

Here u, y denote the input and output; $\theta, \theta_1, \theta_2$ are the disturbances. On the contrary, these examples can be solved easily via the approach proposed in this paper. Moreover, to show a significant application, this paper has derived a successful tracking controller with almost disturbance decoupling for the famous AMIRA ball and beam system.

Throughout the paper, the notation $\|\cdot\|$ denotes the usual Euclidean norm or the corresponding induced matrix norm.

2. Mathematical Model of the AMIRA Ball and Beam System

Figure 1 shows the hardware structure of the AMIRA ball and beam system. U-type aluminium profiles construct the platform and the organization of the ball and beam system which is covered at the side by four sheets of Plexiglas. The steel ball rolls freely on the beam and its position is measured by a camera unit and lighting module mounted below a small platform on top of the system. The beam is located in the center of the system and is driven by a toothbelt, a tooth wheel, and a DC motor. The angle of the beam is measured by an incremental encoder mounted at the rear end of the beam shaft. Two limiting switches are located below the beam to detect whether or not the beam reaches its maximum angle. The unmeasurable states, the speed of the ball and the angular speed of the beam, are estimated via a Luenberger reduced-order observer. Due to the mounting of the beam, the maximum angle is $\alpha_{\max} \approx 0.24$ rad.

Considering all the forces acting upon the system, it is easy to evaluate the kinetic energy, potential energy, dissipative forces, and generalized forces of the system. Inserting them into the Lagrange equation, we obtain the motion equations in higher-order form (Ref. 26),

$$(m + I_b/r^r)\ddot{x}' + \left(mr^2 + I_b\right)(1/r)\ddot{\alpha} - mx'\dot{\alpha}^2 = mg\ \sin\alpha, \tag{1a}$$

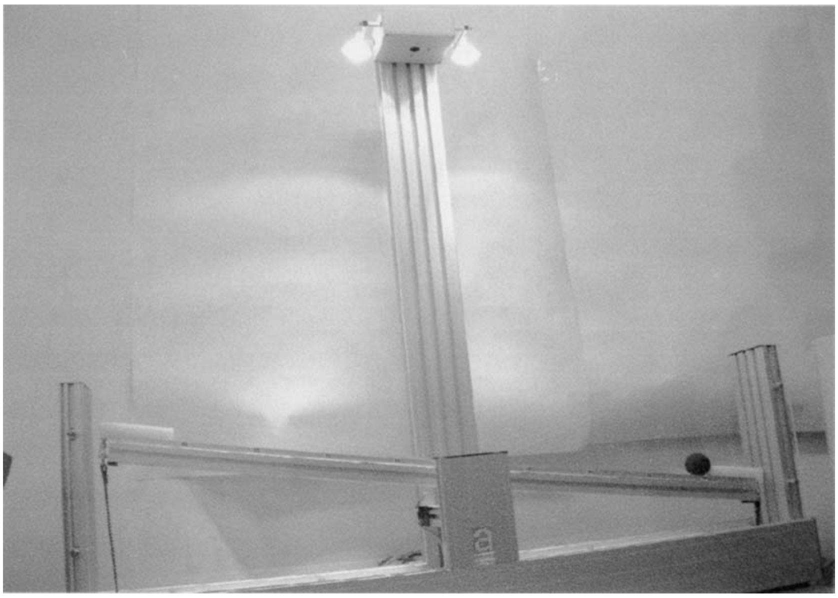

Fig. 1. The AMIRA ball and beam experimental equipment.

$$\left[m(x')^2 + I_b + I_w\right]\ddot{\alpha} + \left(2m\dot{x}'x' + bl^2\right)\dot{\alpha}$$
$$+ kz\, l^2\alpha + \left(mr^2 + I_b\right)(1/r)\ddot{x}' - mgx'\cos\alpha = u\, l\cos\alpha, \tag{1b}$$

and in first-order form,

$$\dot{x}_1 = x_2, \tag{2a}$$
$$\dot{x}_2 = A/B + 0.1\theta_1, \tag{2b}$$
$$\dot{x}_3 = x_4, \tag{2c}$$
$$\dot{x}_4 = C/D - E/F - G/H + (1 + J/K)L/M, \tag{2d}$$

with

$$A = a_2[(b_2x_1x_2 + b_3)x_4 + b_4x_3 - b_6x_1\cos x_3]$$
$$+ \left(mx_1^2 + b_1\right)\left(a_3\sin x_3 + mx_1x_4^2\right) - a_2l\,u\,\cos x_3,$$
$$B = a_1\left(mx_1^2 + b_1\right) - a_2b_5,$$
$$C = -(b_2x_1x_2 + b_3)x_4 - b_4x_3 + b_6x_1\cos x_3,$$
$$D = mx_1^2 + b_1,$$
$$E = b_5[a_3\sin x_3 + mx_1x_4^2],$$

$$F = a_1(mx_1^2 + b_1) - a_2 b_5,$$
$$G = a_2 b_5[(b_2 x_1 x_2 + b_3)x_4 + b_4 x_3 - b_6 x_1 \cos x_3],$$
$$H = \left(mx_1^2 + b_1\right)\left[a_1\left(mx_1^2 + b_1\right) - a_2 b_5\right],$$
$$J = a_2 b_5,$$
$$K = a_1\left(mx_1^2 + b_1\right) - a_2 b_5,$$
$$L = lu \ \cos x_3,$$
$$M = mx_1^2 + b_1,$$

where $m =$ mass of the ball, $r =$ roll radius of the ball, $I_b =$ inertia moment of the ball,

$$a_2 = \left(mr^2 + I_b\right)(1/r), \quad b_2 = 2m,$$

$b =$ friction coefficient of the drive mechanics, $l =$ radius of force application,

$$b_3 = bl^2,$$

$l_w =$ radius of beam, $K =$ stiffness of the drive mechanism, $b_4 = Kl^2$, $g =$ acceleration of gravity, $b_6 = mg$, $I_w =$ inertia moment of the beam, $b_1 = I_b + I_w$, $a_3 = mg$, $u =$ force of the drive mechanics, $a_1 = m + I_b/r^2$, $b_5 = \left(mr^2 + I_b\right)(1/r)$, $x_1 = x' =$ position of the ball, $x_2 = \dot{x}' =$ velocity of the ball, $x_3 = \alpha =$ angle of the beam to the horizontal, $\alpha_{\max} =$ maximum angle of the beam to the horizontal, $x_4 = \alpha =$ angular velocity of the beam; θ_1 is assumed to be the disturbance item. The experimental values adopted are as follows: $r = 0.02$ m, $l = 0.48$ m, $m = 0.0162$ kg, $b = 1.0$ Ns/m, $K = 0.001$ N/m, $l_w = 0.5$ m, $M = 1.122$ kg.

3. Tracking and Almost Disturbance Decoupling Controller Design

In this paper, we consider the following nonlinear control system with disturbances:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \cdot \\ \cdot \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2, \ldots, x_n) \\ f_2(x_1, x_2, \ldots, x_n) \\ \cdot \\ \cdot \\ f_n(x_1, x_2, \ldots, x_n) \end{bmatrix} + \begin{bmatrix} g_1(x_1, x_2, \ldots, x_n) \\ g_2(x_1, x_2, \ldots, x_n) \\ \cdot \\ \cdot \\ g_n(x_1, x_2, \ldots, x_n) \end{bmatrix} u + \sum_{i=1}^{p} q_i^* \theta_i, \tag{3a}$$

$$y(t) = h(x_1, x_2, \ldots, x_n), \tag{3b}$$

that is,

$$\dot{x}(t) = f(x(t)) + g(x(t))u + \sum_{i=1}^{p} q_i^* \theta_i,$$
$$y(t) = h(x(t)),$$

where

$$x(t) := [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \Re^n$$

is the state vector, $u \in \Re^1$ is the input, $y \in \Re^1$ is the output,

$$\theta := [\theta_1(t), \theta_2(t), \ldots, \theta_p(t)]^T$$

is a bounded time-varying disturbance vector, $f, g, q_1^*, \ldots, q_p^*$ are smooth vector fields on $\Re^n$, and $h(x(t)) \in \Re^1$ is a smooth function. The nominal system is then defined as follows:

$$\dot{x}(t) = f(x(t)) + g(x(t))u, \tag{4a}$$
$$y(t) = h(x(t)). \tag{4b}$$

The nominal system (4) consists of the relative degree r (Ref. 27); i.e., there exists a positive integer $1 \leq r < \infty$ such that

$$L_g L_f^k h(x(t)) = 0, \quad k < r-1, \tag{5}$$
$$L_g L_f^{r-1} h(x(t)) \neq 0, \tag{6}$$

for all $x \in \Re^n$ and $t \in [0, \infty)$, where the operator L is the Lie derivative (Ref. 13). The desired output trajectory $y_d(t)$ and its first r derivatives are all uniformly bounded and

$$\left\| \left[y_d(t), y_d^{(1)}(t), \ldots, y_d^{(r)}(t) \right] \right\| \leq B_d, \tag{7}$$

where B_d is some positive constant.

Under the assumption of well-defined relative degree, it has been shown (Ref. 13) that the mapping

$$\phi \Re^n \rightarrow \Re^n \tag{8}$$

defined as

$$\phi_i(x(t)) := \xi_i(t) = L_f^{i-1} h(x(t)), \quad i = 1, 2, \ldots, r, \tag{9}$$
$$\phi_k(x(t)) := \eta_k(t), \quad k = r+1, r+2, \ldots, n, \tag{10}$$

and satisfying

$$L_g \phi_k(x(t)) = 0, \quad k = r+1, r+2, \ldots, n, \tag{11}$$

is a diffeomorphism onto image. For the sake of convenience, define the trajectory error to be

$$e_i(t) := \xi_i(t) - y_d^{(i-1)}(t), \quad i = 1, 2, \ldots, r, \tag{12}$$
$$e(t) := [e_1(t), e_2(t), \ldots, e_r(t)]^T \in \Re^r, \tag{13}$$

the trajectory error multiplied by some adjustable positive constant ε,

$$\bar{e}_i(t) := \varepsilon^{i-1} e_i(t), \quad i = 1, 2, \ldots, r, \tag{14}$$

$$\bar{e}(t) := [\bar{e}_1(t), \bar{e}_2(t), \ldots, \bar{e}_r(t)]^T \in \Re^r, \tag{15}$$

and

$$\xi(t) := [\xi_1(t), \xi_2(t), \ldots, \xi_r(t)]^T \in \Re^r, \tag{16a}$$

$$\eta(t) := \left[\eta_{r+1}(t), \eta_{r+2}(t), \ldots, \eta_n(t)\right]^T \in \Re^{n-r}, \tag{16b}$$

$$\begin{aligned} q(\xi(t), \eta(t)) &:= \left[L_f\phi_{r+1}(t), L_f\phi_{r+2}(t), \ldots, L_f\phi_n(t)\right]^T \\ &:= [q_{r+1} \quad q_{r+2} \quad \cdots \quad q_n]^T. \end{aligned} \tag{16c}$$

Define the phase-variable canonical matrix A_c,

$$A_c := \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ & \vdots & & & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ -\alpha_1 & -\alpha_2 & -\alpha_3 & \ldots & -\alpha_r \end{bmatrix}_{r\times r}, \tag{17}$$

where $\alpha_1, \alpha_2, \ldots, \alpha_r$ are chosen parameters such that A_c is Hurwitz and the vector B is

$$B := [0, 0, \ldots 0, 1]^T{}_{r\times 1}. \tag{18}$$

Let P be the positive-definite solution of the following Lyapunov equation:

$$A_c^T P + P A_c = -I, \tag{19}$$

$$\lambda_{\max}(P) := \text{maximum eigenvalue of } P, \tag{20}$$

$$\lambda_{\min}(P) := \text{minimum eigenvalue of } P, \tag{21}$$

Assumption A1. For all $t \geq 0$, $\eta \in \Re^{n-r}$, and $\xi \in \Re^r$, there exists a positive constant M such that the following inequality holds:

$$\|q_{22}(t, \eta, \bar{e}) - q_{22}(t, \eta, 0)\| \leq M(\|\bar{e}\|), \tag{22}$$

where

$$q_{22}(t, \eta, \bar{e}) := q(\xi, \eta).$$

Assumption A2. There exists a known function $\beta_2(\cdot) : \Re^n \to \Re^+$ such that

$$\left\|K_x^T x\right\| \leq \beta_2 \|\bar{e}\|, \tag{23}$$

where $K_x := [k_{x1}, k_{x2}, \ldots, k_{xn}]^T_{n\times 1}$ and $k_{xi}, i = 1, 2, \ldots, n$, are real constants.

For the sake of stating precisely the problem investigated, define

$$d := L_g L_f^{r-1} h(x(t)), \quad c := L_f^r h(x(t)), \tag{24a}$$

and

$$\bar{\bar{e}} := \alpha_1 \bar{e}_1 + \alpha_2 \bar{e}_2 + \cdots + \alpha_r \bar{e}_r. \tag{24b}$$

Definition 3.1. See Ref. 2. A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class K if it is strictly increasing and $\alpha(0) = 0$.

Definition 3.2. See Ref. 2. A continuous function $\beta : [0, a) \times [0, \infty) \to [0, \infty)$ is said to belong to class KL if, for each fixed s, the mapping $\beta(r, s)$ belongs to class K with respect to r and , for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ and $s \to \infty$.

Definition 3.3. See Ref. 2. Consider the system $\dot{x} = f(t, x, \theta)$, where $f : [0, \infty) \times \Re^n \times \Re^n \to \Re^n$ is piecewise continuous in t and locally Lipschitz in x and θ. This system is said to be input-to-state stable if there exists a class KL function β, a class K function γ, and positive constants k_1 and k_2 such that, for any initial state $x(t_0)$ with $\|x(t_0)\| < k_1$ and for any bounded input $\theta(t)$ with $\sup_{t \geq t_0} \|\theta(t)\| < k_2$, the state exists and satisfies

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t - t_0) + \gamma\left(\sup_{t_0 \leq \tau \leq t} \|\theta(\tau)\|\right), \tag{25a}$$

for all $t \geq t_0 \geq 0$.

Now, we formulate the almost disturbance decoupling problem as follows.

Definition 3.4. See Ref. 24. The tracking problem with almost disturbance decoupling is said to be globally solvable by the state feedback controller u for the transformed error system by a global diffeomorphism (8) if the controller u enjoys the following properties:

(i) It is input-to-state with respect to disturbance inputs.
(ii) For any initial value $\bar{x}_{e0} := [\bar{e}(t_0), \eta(t_0)]^T$, for any $t \geq t_0$, and for any $t_0 \geq 0$,

$$|y(t) - y_d(t)| \leq \beta_{11}(\|x(t_0)\|, t - t_0) + \left(1/\sqrt{\beta_{22}}\right)\beta_{33}\left(\sup_{t_0 \leq \tau \leq t} \|\theta(\tau)\|\right), \tag{25b}$$

$$\int_{t_0}^{t} [y(\tau) - y_d(\tau)]^2 d\tau \leq (1/\beta_{44})\left[\beta_{55}(\|\bar{x}_{e0}\|) + \int_{t_0}^{t} \beta_{33}\left(\|\theta(\tau)\|^2\right) d\tau\right], \tag{25c}$$

where β_{22}, β_{44} are some positive constants, β_{33}, β_{55} are class K functions, and β_{11} is a class KL function.

Theorem 3.1. Suppose that there exists a continuously differentiable function $V : \Re^{n-r} \to \Re^+$ such that the following three inequalities hold for all $\eta \in \Re^{n-r}$:

$$\text{(a) } \omega_1 \|\eta\|^2 \leq V(\eta) \leq \omega_2 \|\eta\|^2, \quad \omega_1, \omega_2 > 0, \tag{26a}$$

$$\text{(b) } \nabla_t V + (\nabla_\eta V)^T q_{22}(t, \eta, 0) \leq -2\alpha_x \|\eta\|^2, \quad \alpha_x > 0, \tag{26b}$$

$$\text{(c) } \left\|\nabla_\eta V\right\| \leq \varpi_3 \|\eta\|, \quad \varpi_3 > 0. \tag{26c}$$

Then, the tracking problem with almost disturbance decoupling is globally solvable by the controller defined by

$$\begin{aligned} u = &\left[L_g L_f^{r-1} h(x(t))\right]^{-1} \{-L_f^r h(x) + y_d^{(r)} - \varepsilon^{-r}\alpha_1 \left[L_f^0 h(x) - y_d\right] \\ &- \varepsilon^{1-r}\alpha_2 \left[L_f^1 h(x) - y_d^{(1)}\right] - \cdots - \varepsilon^{-1}\alpha_r \left[L_f^{r-1} h(x) - y_d^{(r-1)}\right] + K_x^T X\}, \end{aligned} \tag{27}$$

where $K := [k_1, k_2, \cdots, k_n]$ is some adjustable real matrix. Moreover, the influence of disturbances on the L_2 norm of the tracking error can be attenuated arbitrarily by increasing the value of the following adjustable parameter $NN_2 > 1$:

$$\begin{aligned} H(\varepsilon) &:= \begin{bmatrix} H_{11} & H_{12} \\ H_{12} & H_{22} \end{bmatrix} \\ &:= \begin{bmatrix} 2\alpha_x - (\omega_3^2/\omega_1)\|\phi_\eta\|^2 & -\left(1/\sqrt{k(\varepsilon)}\right)\left[w_3 M/\sqrt{2w_1 \lambda_{\min}(P)}\right] \\ -\left(1/\sqrt{k(\varepsilon)}\right)\left[w_3 M/\sqrt{2w_1 \lambda_{\min}(P)}\right] & (1/\varepsilon\lambda_{\max}(P)) - \left[2\varepsilon^r \|B^T P\| \beta_2 + 2(k(\varepsilon)/\varepsilon)\|\phi_\xi\|^2 \cdot \|P\|^2/\varepsilon\lambda_{\min}(P)\right] \end{bmatrix} \end{aligned} \tag{28a}$$

$$\alpha_s(\varepsilon) := \left\{ H_{11} + H_{22} - \left[(H_{11} - H_{22})^2 + 4H_{22}^2\right]^{1/2} \right\} \Big/ 4, \tag{28b}$$

$$N := 2\alpha_s(\varepsilon), \tag{28c}$$

$$N_2 := \min\{\omega_1, [\bar{k}(\varepsilon)/2]\lambda_{\min}(P)\}, \tag{28d}$$

$$\phi_\xi(\varepsilon) := \begin{bmatrix} \varepsilon(\partial/\partial X)hq_1^* & \cdots & \varepsilon(\partial/\partial X)hq_p^* \\ \vdots & & \vdots \\ \varepsilon^r(\partial/\partial X)L_f^{r-1}hq_1^* & \cdots & \varepsilon^r(\partial/\partial X)L_f^{r-1}hq_q^* \end{bmatrix}, \tag{28e}$$

$$\phi_\eta(\varepsilon) := \begin{bmatrix} (\partial/\partial X)\phi_{r+1}q_1^* & \cdots & (\partial/\partial X)\phi_{r+1}q_p^* \\ \vdots & & \vdots \\ (\partial/\partial X)\phi_n q_1^* & \cdots & (\partial/\partial X)\phi_n q_q^* \end{bmatrix}, \tag{28f}$$

where H is a positive-definite matrix and $k(\varepsilon) : \Re^+ \to \Re^+$ is any continuous function satisfying

$$\lim_{\varepsilon\to o} k(\varepsilon) = 0 \quad \text{and} \quad \lim_{\varepsilon\to o}[\varepsilon/k(\varepsilon)] = 0. \tag{28g}$$

Proof. Applying the coordinate transformation (8) yields

$$\begin{aligned}
\dot{\xi}_1(t) &= (\partial\phi_1/\partial x)(dx/dt) \\
&= [\partial h(x(t))/\partial x]\left[f + g\cdot u + \sum_{i=1}^{p} q_i^*\theta_i\right] \\
&= L_f^1 h(x(t)) + [\partial h(x)/\partial x]\sum_{i=1}^{p} q_i^*\theta_i \\
&= \xi_2(t) + \sum_{i=1}^{p}[\partial h(x)/\partial x]q_i^*\theta_i,
\end{aligned} \tag{29}$$

$$\vdots$$

$$\begin{aligned}
\dot{\xi}_{r-1}(t) &= (\partial\phi_{r-1}/\partial x)(dx/dt) \\
&= \partial L_f^{r-2} h(x(t))/\partial x\left[f + g\cdot u + \sum_{i=1}^{p} q_i^*\theta_i\right] \\
&= L_f^{r-1} h(x(t)) + \left[\partial L_f^{r-2} h(x(t))/\partial x\right]\sum_{i=1}^{p} q_i^*\theta_i \\
&= \xi_r(t) + \sum_{i=1}^{p}\left[\partial L_f^{r-2} h(x(t))/\partial x\right]q_i^*\theta_i,
\end{aligned} \tag{30}$$

$$\begin{aligned}
\dot{\xi}_r(t) &= (\partial\phi_r/\partial x)(dx/dt) \\
&= \partial L_f^{r-1} h(x(t))/\partial x\left[f + g\cdot u + \sum_{i=1}^{p} q_i^*\theta_i\right] \\
&= L_f^r h(x) + L_g L_f^{r-1} h(x)u + \sum_{i=1}^{p}\left[\partial L_f^{r-1} h(x(t))/\partial x\right]q_i^*\theta_i, \\
&= c + du + \sum_{i=1}^{p}\left[\partial L_f^{r-1} h(x(t))/\partial x\right]q_i^*\theta_i,
\end{aligned} \tag{31}$$

$$\begin{aligned}
\dot{\eta}_k(t) &= [\partial\phi_k(x)/\partial x][dx/dt] \\
&= [\partial\phi_k(x)/\partial x]\left[f + g\cdot u + \sum_{i=1}^{p} q_i^*\theta_i\right] \\
&= L_f\phi_k + \sum_{i=1}^{p}[\partial\phi_k(x)/\partial x]q_i^*\theta_i, \quad k = r+1, r+2, \ldots, n.
\end{aligned} \tag{32}$$

Since

$$c(\xi(t), \eta(t)) := L_f^r h(x(t)), \tag{33}$$

$$d(\xi(t), \eta(t) := L_g L_f^{r-1} h(x(t)), \tag{34}$$

$$q_k(\xi(t), \eta(t)) = L_f \phi_k(x), \quad k = r+1, r+2, \ldots, n, \tag{35}$$

the dynamic equations of the system (3) in the new coordinates are as follows:

$$\dot{\zeta}(t) = \xi_{i+1}(t) + \sum_{i=1}^{p} (\partial/\partial X) L_f^{i-1} h q_i^* \theta_i, \quad i = 1, 2, \ldots, r-1, \tag{36}$$

$$\dot{\xi}_r(t) = c(\xi(t), \eta(t)) + d(\xi(t), \eta(t)) u + \sum_{i=1}^{p} (\partial/\partial X) L_f^{r-1} h q_i^* \theta_i, \tag{37}$$

$$\dot{\eta}_k(t) = q_k(\xi(t), \eta(t)) + \sum_{i=1}^{p} (\partial/\partial x) \phi_k(X) q_i^* \theta_i, \quad k = r+1, \ldots, n, \tag{38}$$

$$y(t) = \xi_1(t). \tag{39}$$

Define

$$\begin{aligned} v := y_d^{(r)} - \varepsilon^{-r}\alpha_1 \left[L_f^0 h(x) - y_d\right] - \varepsilon^{1-r}\alpha_2 \left[L_f^1 h(x) - y_d^{(1)}\right] \\ - \cdots - \varepsilon^{-1}\alpha_r \left[L_f^{r-1} h(x) - y_d^{(r-1)}\right] + K_x^T x. \end{aligned} \tag{40}$$

According to equations (9), (12), (33), (34), (40), the tracking controller can be rewritten as

$$u = d^{-1}[-c + v]. \tag{41}$$

Substituting equation (41) into (37), the dynamic equations of the system (3) can be shown as follows:

$$\begin{bmatrix} \dot{\xi}_1(t) \\ \dot{\xi}_2(t) \\ \vdots \\ \dot{\xi}_{r-1}(t) \\ \dot{\xi}_r(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \dot{\xi}_1(t) \\ \dot{\xi}_2(t) \\ \vdots \\ \dot{\xi}_{r-1}(t) \\ \dot{\xi}_r(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} v + \begin{bmatrix} \sum_{i=1}^{p} (\partial/\partial x) h q_i^* \theta_i \\ \sum_{i=1}^{p} (\partial/\partial x) L_f^1 h q_i^* \theta_i \\ \vdots \\ \sum_{i=1}^{p} (\partial/\partial x) L_f^{r-1} h q_i^* \theta_i \end{bmatrix}, \tag{42}$$

$$\begin{bmatrix} \dot{\eta}_{r+1}(t) \\ \dot{\eta}_{r+2}(t) \\ \vdots \\ \dot{\eta}_{n-1}(t) \\ \dot{\eta}_n(t) \end{bmatrix} = \begin{bmatrix} q_{r+1}(t) \\ q_{r+2}(t) \\ \vdots \\ q_{n-1}(t) \\ q_n(t) \end{bmatrix} + \begin{bmatrix} \sum_{i=1}^{p}(\partial/\partial x)\phi_{r+1}q_i^*\theta_i \\ \sum_{i=1}^{p}(\partial/\partial x)\phi_{r+2}q_i^*\theta_i \\ \vdots \\ \sum_{i=1}^{p}(\partial/\partial x)\phi_{n-1}q_i^*\theta_i \\ \sum_{i=1}^{p}(\partial/\partial x)\phi_n q_i^*\theta_i \end{bmatrix}, \tag{43}$$

$$y = [1\,0 \cdots 0\,0]_{r\times 1} \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \\ \vdots \\ \xi_{r-1}(t) \\ \xi_r(t) \end{bmatrix}_{r\times 1} = \xi_1(t). \tag{44}$$

Combining equations (12), (14), (17), (40), it can be verified easily that equations (42)–(44) can be transformed into the following form, which is organized as a singularly-perturbed system (Refs. 28–29):

$$\dot{\eta}(t) = q(\xi(t), \eta(t)) + \phi_\eta \theta$$

$$:= q_{22}(t, \eta(t), \bar{e}) + \phi_\eta \theta, \tag{45a}$$

$$\varepsilon\dot{\bar{e}}(t) = A_c\bar{e} + B\varepsilon^r K_x^T x + \phi_\xi \theta, \tag{45b}$$

$$y(t) = \xi_1(t). \tag{46}$$

We consider $L(\bar{e}, \eta)$ defined by a weighted sum of $V(\eta)$ and $W(\bar{e})$,

$$L(\bar{e}, \eta) := V(\eta) + k(\varepsilon)W(\bar{e}), \tag{47}$$

as a composite Lyapunov function of the subsystems (45) Refs. 30–31, where $W(e)$ satisfies

$$W(\bar{e}) := (1/2)\bar{e}^T P \bar{e}. \tag{48}$$

In view of (19), (22), (23), (26), (27), the derivative of L along the trajectories of (45) is given by

$$
\begin{aligned}
\dot{L} &= \left[\nabla_t V + \left(\nabla_\eta V\right)^T \dot{\eta}\right] + (k/2)\left[\left(\dot{\bar{e}}\right)^T P \bar{e} + \bar{e}^T P \left(\dot{\bar{e}}\right)\right] \\
&= \left[\nabla_t V + \left(\nabla_\eta V\right)^T \dot{\eta}\right] + (K/2\varepsilon)\left[\dot{\bar{e}}^T \left(A_c^T P + P A_c\right)\bar{e}\right] \\
&\quad + (k/2)\varepsilon^{r-1}\left\{2B^T P \bar{e}\left(K_x^T x\right)\right\} + k\left\{(1/\varepsilon)\theta^T \phi_\xi^T P \bar{e}\right\} \\
&\le \left[\nabla_t V + \left(\nabla_\eta V\right)^T q_{22}(t,\eta(t),0)\right] \\
&\quad + \left\|\nabla_\eta V\right\| \left\|q_{22}(t,\eta(t),\bar{e}) - q_{22}(t,\eta(t),0)\right\| \\
&\quad + \left\|\nabla_\eta V\right\| \left\|\phi_\eta\right\| \|\theta\| - [k/\varepsilon\lambda_{\max}(P)]W + k\varepsilon^{r-1}\left\{\left\|B^T P\right\| \|\bar{e}\| \left\|K_x^T X\right\|\right\} \\
&\quad + (k/\varepsilon)\|\theta\| \left\|\phi_\xi\right\| \|P\| \|\bar{e}\| \\
&\le -2\alpha_x V + \omega_3 \|\eta\| M \|\bar{e}\| + \omega_3 \|\eta\| \left\|\phi_\eta\right\| \|\theta\| - [k/\varepsilon\lambda_{\max}(P)]W \\
&\quad + k\varepsilon^{r-1}\left\|B^T P\right\| \beta_2[2W/\lambda_{\min}(P)] \\
&\quad + (k/\varepsilon)\|\theta\| \left\|\phi_\xi\right\| \|P\| \|\bar{e}\| \\
&\le -\left(2\alpha_x - (\omega_3^2/\omega_1)\left\|\phi_\eta\right\|^2\right)V + 2\left(W_3 M/\sqrt{2W_1\lambda_{\min}(P)}\right)\sqrt{V}\sqrt{W} \\
&\quad - \left(k/\varepsilon\lambda_{\max}(P) - \varepsilon^{r-1}\left(k\left\|B^T P\right\|\beta_2/(1/2)\lambda_{\min}(P)\right.\right. \\
&\quad \left.\left. - k^2\left\|\phi_\xi\right\|^2 \|P\|^2/\left(\varepsilon^2/2\right)\lambda_{\min}(P)\right)W + (1/2)\|\theta\|^2 \\
&= -\left[\sqrt{V}, \sqrt{kW}\right] H \begin{bmatrix} \sqrt{V} \\ \sqrt{kW} \end{bmatrix} + (1/2)\|\theta\|^2,
\end{aligned}
\tag{49}
$$

i.e.,

$$
\dot{L} \le -\lambda_{\min}(H)L + (1/2)\|\theta\|^2, \tag{50}
$$

where $\lambda_{\min}(H)$ denotes the minimum eigenvalue of the matrix H. Utilizing the fact that

$$
\lambda_{\min}(H) = 2\alpha,
$$

We obtain

$$
\begin{aligned}
\dot{L} &\le -2\alpha_s L + (1/2)\|\theta\|^2 \\
&\le -2\alpha_s (V + kW) + (1/2)\|\theta\|^2 \\
&\le -2\alpha_s \left(\omega_1 \|\eta\|^2 + (k/2)\lambda_{\min}(P)\|\bar{e}\|^2\right) + (1/2)\|\theta\|^2 \\
&\le -NN_2\left(\|\eta\|^2 + \|\bar{e}\|^2\right) + (1/2)\|\theta\|^2.
\end{aligned}
\tag{51}
$$

Define

$$
\bar{e}_{1r} := \begin{bmatrix} \bar{e}_2 \\ \vdots \\ \bar{e}_r \end{bmatrix}. \tag{52}
$$

Hence,

$$\dot{L} \leq -NN_2\left(\|\eta\|^2 + \|\bar{e}_1\|^2 + \|\bar{e}_{1r}\|^2\right) + (1/2)\|\theta\|^2. \tag{53}$$

Utilizing (53) yields easily

$$\int_{t_0}^{t} (y(\tau) - y_d(\tau))^2 d\tau \leq L(t_0)/NN_2 + (1/2NN_2)\int_{t_0}^{t} \|\theta(\tau)\|^2 d\tau, \tag{54}$$

so that the statement (25c) is satisfied. From (51), we get

$$\dot{L} \leq -NN_2\left(\|y_{\text{total}}\|^2\right) + (1/2)\|\theta\|^2, \tag{55a}$$

where

$$\|y_{\text{total}}\|^2 := \|\bar{e}\|^2 + \|\eta\|^2. \tag{55b}$$

By virtue of Ref. 2, Theorem 5.2, (55a) implies the input-to-state stability for the closed-loop system. Furthermore, it is easy to see that

$$\Delta_{\min}\left(\|\bar{e}\|^2 + \|\eta\|^2\right) \leq L \leq \Delta_{\max}\left(\|\bar{e}\|^2 + \|\eta\|^2\right); \tag{56}$$

i.e.,

$$\Delta_{\min}\left(\|y_{\text{total}}\|^2\right) \leq L \leq \Delta_{\max}\left(\|y_{\text{total}}\|^2\right), \tag{57}$$

where

$$\Delta_{\min} := \min\{w_1, (k/2)\lambda_{\min}(P)\},$$
$$\Delta_{\max} := \min\{w_2, (k/2)\lambda_{\max}(P)\}.$$

From (51) and (57), we get

$$\dot{L} \leq -(NN_2/\Delta_{\max})L + (1/2)\left(\sup_{t_0 \leq \tau \leq t} \|\theta(\tau)\|\right)^2. \tag{58}$$

Hence,

$$\begin{aligned} L(t) \leq\ & L(t_0)\exp\left[-(NN_2/\Delta_{\max})(t - t_0)\right] \\ & + (\Delta_{\max}/2NN_2)\left(\sup_{t_0 \leq \tau \leq t} \|\theta(\tau)\|\right)^2, \quad t \geqslant t_0, \end{aligned} \tag{59}$$

which implies

$$\begin{aligned} |e_1(t)| \leq\ & \sqrt{2L(t_0)/k\lambda_{\min}(P)}\exp\left[-(NN_2/2\Delta_{\max})(t - t_0)\right] \\ & + \sqrt{\Delta_{\max}/k\lambda_{\min}(P)NN_2}\left(\sup_{t_0 \leq \tau \leq t} \|\theta(\tau)\|\right), \end{aligned} \tag{60}$$

so that the statement (25b) is proved and then the tracking problem with almost disturbance decoupling is globally solved.

Applying Theorem 3.1 to the AMIRA ball and beam system now, we have completed some experiments and achieved the almost disturbance decoupling performance and the goal of finding a tracking controller u that steers the angle of beam x_3 and the position of ball x_1, starting from any initial values, to track the desired zero function (i.e. $y_d = 0$). In order to achieve the goal, we choose

$$h(X) = x_1 + x_2 + x_3 + x_4.$$

Based on the constraint of hardware, $h(X) \to 0$ implies $x_1 \to 0$ and $x_3 \to 0$. Let us choose arbitrarily $\alpha_1 = 0.007$ and $K_x = [-0.7, -0.7, -0.7, -0.7]^T$ such that $A_c = -0.007$ is Hurwitz and $P = 71.43$. The AMIRA ball and beam system is a system of relative degree one. It can be verified that, with the choice

$$V(\eta) = \eta_2^2 + \eta_3^2 + \eta_4^2,$$

conditions (26) and (28) are satisfied with

$$\varepsilon = 0.0025, \quad \beta_2 = 0.7, \quad M = \sqrt{3}, \quad \omega_1 = 1, \quad \omega_2 = 1, \quad \alpha_x = 1,$$
$$\omega_3 = 2, \quad H_{11} = 2, \quad H_{12} = -1.296, \quad H_{22} = 3.8, \quad N = 1.325, \quad N_2 = 1.$$

From (27), we obtain the desired tracking controller

$$\begin{aligned} u = (b_{11}/b_{22} + a)^{-1}\{&-2.8x_1 - 3.8x_2 - 3.5x_3 - 4.5x_4 \\ &- (a_{12} + a_{34})/a_5 - (z_{12} + z_{34} + z_{56})/z_7\}. \end{aligned} \tag{61}$$

where

$$\begin{aligned} a_{12} = {} & 0.0000561038x_1x_2x_4 + 0.000415.7x_4 \\ & + 0.000000415757x_3 - 0.00027490882x_1 \cos x_3 \end{aligned}$$

$$\begin{aligned} a_{34} = {} & 0.0025719x_1^2 \sin x_3 + 0.00026244x_1^3x_4^2 + 0.03711015 \sin x_3 \\ & + 0.00378675x_1x_4^2 \end{aligned}$$

$$a_5 = 0.0015x_1^2 + 0.021642252$$

$$\begin{aligned} z_{12} = {} & -0.0000486x_1^3x_2x_4 - 0.00036018x_1^2x_4 - 0.00000036x_1^2x_3 \\ & + 0.000238x_1^3 \cos x_3 \end{aligned}$$

$$\begin{aligned} z_{34} = {} & -0.0007013x_1x_2x_4 - 0.005197x_4 - 0.000005197x_3 \\ & + 0.003436x_1 \cos x_3 \end{aligned}$$

$$\begin{aligned} z_{56} = {} & -0.00000445x_1^2 \sin x_3 - 0.000000454x_1^3x_4^2 - 0.0000643 \sin x_3 \\ & - 0.00000655x_1x_4^2 \end{aligned}$$

$$z_7 = 0.0000243x_1^4 + 0.00070126x_1^2 + 0.021645$$

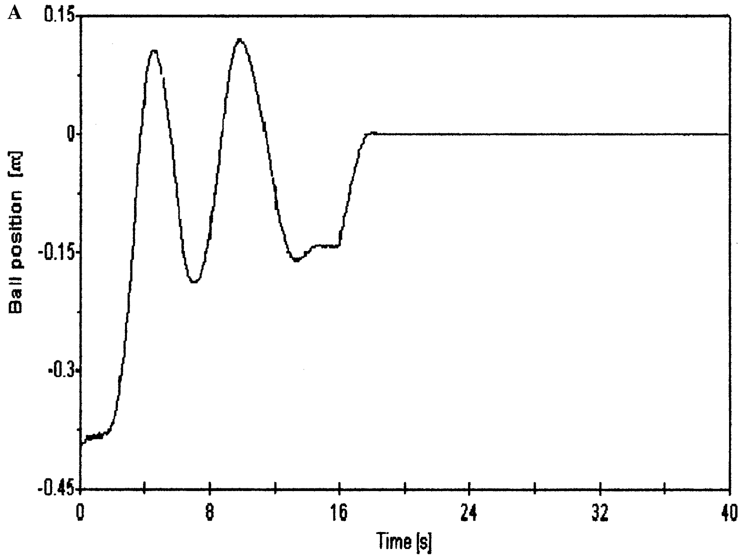

Fig. 2A. Position of ball for AMIRA ball and beam system without disturbance.

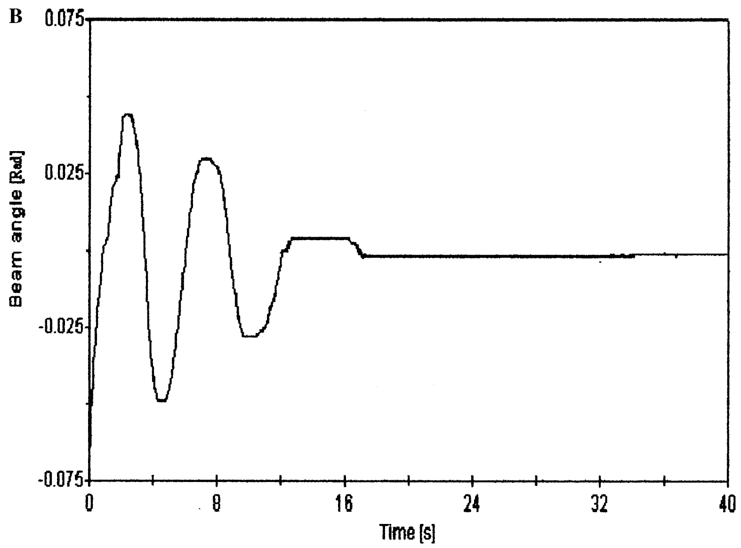

Fig. 2B. Angle of beam for AMIRA ball and beam system without disturbance.

A

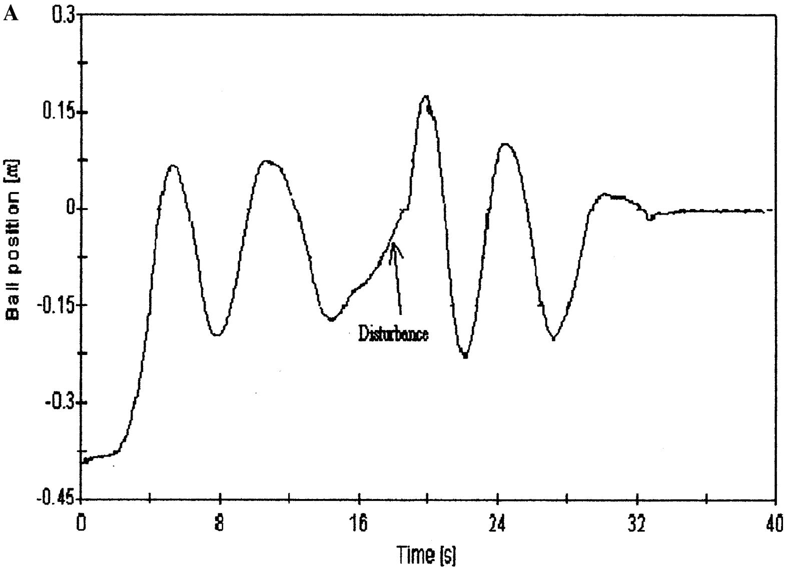

Fig. 3A. Position of ball for AMIRA ball and beam system with disturbance.

B

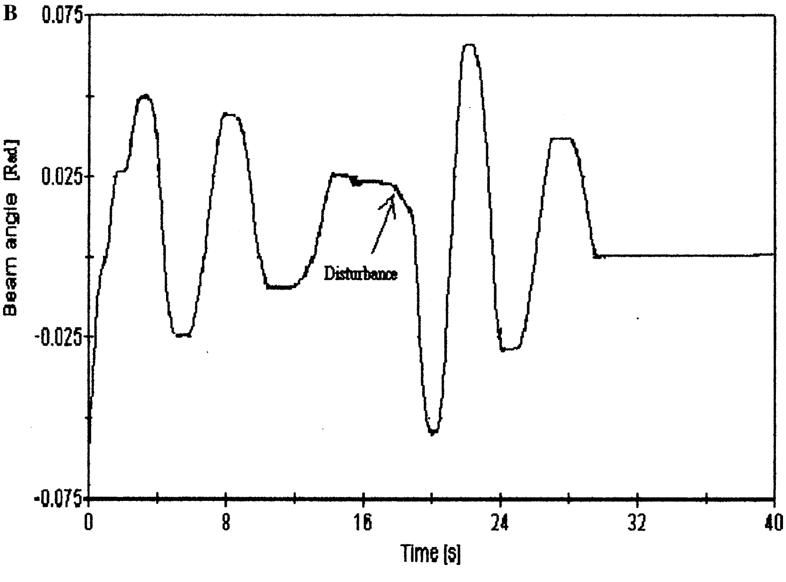

Fig. 3B. Angle of beam for AMIRA ball and beam system with disturbance.

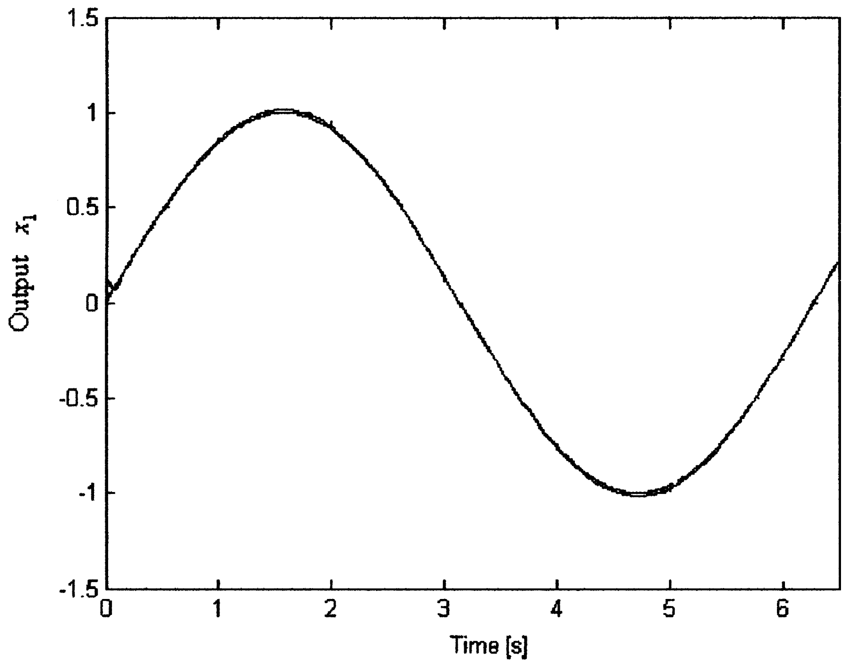

Fig. 4. Output trajectory of feedback-controlled system for (62).

Hence, the tracking controller will steer the angle of beam x_3 and the position of ball x_1 to track the desired trajectory

$$y_d(t) = h(x) = x_1 + x_2 + x_3 + x_4 = 0$$

in view of Theorem 3.1. Figures 2 and 3 show the experimental results based on our proposed tracking controller without and with disturbance respectively. It is clear that the tracking controller acheives the stable tracking and the almost disturbance decoupling performance.

4. Comparison with Existing Approaches

Reference 1 exploits the fact that, for nonlinear single-input single-output systems, the almost disturbance decoupling problem cannot be solved, as the following example shows:

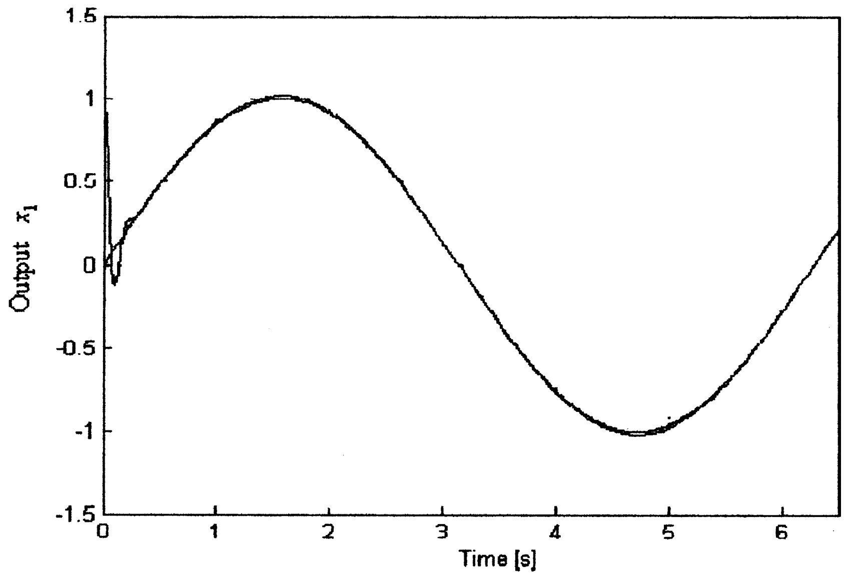

Fig. 5. Output trajectory of feedback-controlled system for (64).

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} \tan^{-1} x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \theta(t), \tag{62a}$$

$$y(t) = x_1(t) := h(x(t)), \tag{62b}$$

where u, y denote the input and output respectively, $\theta(t) := 0.5 \sin t$ is the disturbance. On the contrary, this problem can be solved easily via the approach proposed in this paper. Following the same procedures shown in the demonstrated example, the tracking problem with almost disturbance decoupling problem can be solved via the state feedback controller u defined as

$$u = \left(1 + x_2^2\right)[-\sin t - (0.03)^{-2}(x_1 - \sin t) - (0.03)^{-1}(\tan^{-1} x_2 - \cos t)]. \tag{63}$$

The output trajectory of the feedback-controlled system for (62) is shown in Fig. 4 via MATLAB. It is also shown in Ref. 1 by the following example:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = + \begin{bmatrix} x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} \theta_1(t) \\ x_2^3 \theta_2(t) \end{bmatrix}, \quad \theta_1(t) = \theta_2(t) = 0.5 \sin t, \tag{64a}$$

$$y(t) = x_1(t) := h(X(t)) \tag{64b}$$

that the tracking and almost disturbance decoupling problem is not achieved. The feedback control algorithm proposed in this paper solves it perfectly. Applying the same design procedures of Theorem 3.1 yields the desired tracking and almost disturbance decoupling controller as follows:

$$u = -\sin t - (0.03)^{-2}(x_1 - \sin t) + (0.03)^{-1}(x_2 - \cos t). \tag{65}$$

The output trajectory of the feedback-controlled system for (64) is shown in Fig. 5 via MATLAB. From Figs. 4–5, it is obvious that the desired tracking and almost disturbance decoupling performance are achieved.

5. Conclusions

In this paper, we have constructed a feedback control algorithm which globally solves the tracking problem with almost disturbance decoupling for the AMIRA ball and beam system. The discussion and practical application of feedback linearization of nonlinear control systems by parametrized coordinate transformation have been presented. Two comparative examples are proposed to show the significant contribution of this paper with respect to existing approaches. Moreover, a practical example of the AMIRA ball and beam system demonstrates the applicability of the proposed differential geometry approach and the composite Lyapunov approach. Simulation results exploited the fact that the proposed methodology is successfully applied to feedback linearization problem and achieves the desired tracking and almost disturbance decoupling performances of the controlled system.

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